Registration No.: | | | | | | | | | | |

Total Number of Pages: 02 **Course: M.Sc.I**

Sub_Code: FMCC1001

10th Semester Regular Examination: 2024-25
SUBJECT: DIFFERENTIAL GEOMETRY
BRANCH(S): M.Sc.I(MC)
Time: 3 Hours
Max Marks: 70
Q.Code: S051
Answer Question No.1 (Part-I) which is compulsory, any five from rest (Part-II)
The figures in the right hand margin indicate marks.

Part-I

Q1 **Answer the following questions :** **(2 x 10)**

a) Consider a system of equations $T: \begin{cases} y^1 = x^1 \sin x^2 \cos x^3, \\ y^2 = x^1 \sin x^2 \sin x^3, \\ y^3 = x^1 \cos x^2, \end{cases}$ specifying the relation between the spherical polar coordinates x^i and the rectangular cartesian coordinates y^i. Find the inverse transformation of the given system.

b) Explain mixed tensor with a suitable example.

c) Consider the tensors $A_{ij}(x), A_k(x)$, and $A^k(x)$. The outer product is given by$A_{ij}A_k = A^k$. What is the rank of the covariant tensor and state whether contraction is possible?

d) Define symmetric and skew-symmetric tensors.

e) Define curvilinear coordinates and state the reason for the terminology.

f) State the reason for which geodesics in E_n are straight lines.

g) Write the definition of intrinsic or absolute derivative.

h) Define geodesic curvature of a surface curve C.

i) Give an example of surfaces that are isometric in the Euclidean plane.

j) Show that the Gaussian curvature is invariant.

Part-II

Long Answer Type Questions (Answer Any five)

Q2 **a)** Prove that the set of all admissible transformations of coordinates forms a group. **(5 + 5)**

b) Write a short note on metric tensor.

Q3 **a)** Prove that the sum of two tensors which have the same number of covariant and contravariant indices is again a tensor of the same type and ranks as the given tensors.

b) Describe Christoffel's symbols and their transformation laws.

Q4 a) State and prove the necessary and sufficient condition for the metric coefficients $g_{ij}(x)$ reduce to constants h_{ij} in some reference frame Y. **(5 + 5)**

b) Consider a curve, defined in cylindrical coordinates by equations $\begin{cases} x^1 = a, \\ x^2 = \theta(s), \\ x^3 = 0. \end{cases}$ Show that the torsion $\tau = 0$.

Q5 a) State and prove the necessary and sufficient condition for a given curvilinear coordinate system X to be orthogonal. **(5 + 5)**

b) Determine the equation of geodesics on an arbitrary cylinder immersed in E_3.

Q6 a) Prove that a tangent vector is a linear mapping of $\mathcal{D}^{\infty}(P)$ into the real numbers which is also a differentiation. **(5 + 5)**

b) State and prove the necessary and sufficient condition that a curve on a surface S be geodesic

Q7 a) State and derive the Weingarten formula. **(5 + 5)**

b) Derive the Gauss and Codazzi equation.

Q8 a) State and prove the Meusnier's theorem. **(5 + 5)**

b) Write a short note on first and second fundamental tensor.